If $a \in U$, $b \in W$, then write $a = 3k_1$, and $b = 3k_2 + 2$, and take $c_1 = 3k_1 + 1$ and $c_2 = 3k_2 + 1$ as the required elements in V.

If $a \in V$, $b \in W$, then write $a = 3k_1 + 1$ and $b = 3k_2 + 2$, and take $c_1 = 3k_1$ and $c_2 = 3(k_2 + 1)$ as the required elements in U.

Therefore, U, V, and W satisfy the prescribed condition.

3. (V. Karamzin) Let a, b, and c be positive real numbers such that $abc = 1$. Prove that $2\left(a^2 + b^2 + c^2\right) + a + b + c \geq ab + bc + ca + 6$.

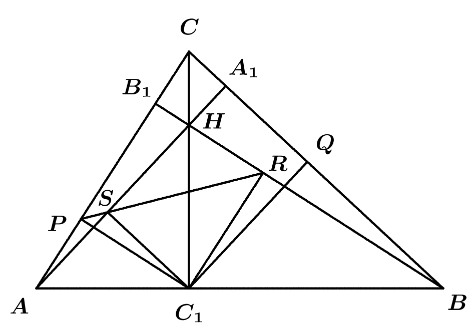
Solved by Arkady Alt, San Jose, CA, USA; George Apostolopoulos, Messolonghi, Greece; Michel Bataille, Rouen, France; and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, ON. We give Alt's version.

Since $a + b + c \geq 3\sqrt[3]{abc} = 3$ and $ab + bc + ca \geq 3\sqrt[3]{a^2b^2c^2} = 3$ by the AM–GM Inequality, then we have

$$\begin{aligned}
&2\left(a^2 + b^2 + c^2\right) + a + b + c - (ab + bc + ca) - 6 \\
&= 2\left(a^2 + b^2 + c^2 - ab - bc - ca\right) + a + b + c + ab + bc + ca - 6 \\
&= (a-b)^2 + (b-c)^2 + (c-a)^2 \\
&\qquad + (a + b + c - 3) + (ab + bc + ca - 3) \geq 0\,.
\end{aligned}$$

5. (I. Voronovich) Let AA_1, BB_1, and CC_1 be the altitudes of an acute triangle ABC. Prove that the feet of the perpendiculars from C_1 to the segments AC, BC, BB_1, and AA_1 are collinear.

Solved by Miguel Amengual Covas, Cala Figuera, Mallorca, Spain; Michel Bataille, Rouen, France; Geoffrey A. Kandall, Hamden, CT, USA; and Titu Zvonaru, Comăneşti, Romania. We give Kandall's version.



Let P, Q, R, S be the feet of the perpendiculars from C_1 to AC, BC, BB_1, AA_1, respectively, and let the orthocentre of ABC be H. Draw PS and SR.

The quadrilaterals $APSC_1$ and $SHRC_1$ are cyclic, and so $\angle PSA = \angle PC_1A = 90^\circ - \angle CAB$ and $\angle HSR = \angle HC_1R = 90^\circ - \angle RC_1B = \angle RBA = 90^\circ - \angle CAB$. Thus, $\angle PSA = \angle HSR$, that is, the points P, S, and R are collinear. The proof that S, R, and Q are collinear is analogous. Therefore, P, S, R, and Q are collinear.

7. (I. Zhuk) Let x, y, and z be real numbers greater than 1 such that

$$\begin{aligned} xy^2 - y^2 + 4xy + 4x - 4y &= 4004\,, \\ xz^2 - z^2 + 6xz + 9x - 6z &= 1009\,. \end{aligned}$$

Determine all possible values of $xyz + 3xy + 2xz - yz + 6x - 3y - 2z$.

Solved by Arkady Alt, San Jose, CA, USA; Konstantine Zelator, University of Pittsburgh, Pittsburgh, PA, USA; and Titu Zvonaru, Comăneşti, Romania. We give Zelator's solution.

The first equation is equivalent to $x(y^2 + 4y + 4) = 4004 + y^2 + 4y$, or $x(y+2)^2 = 4000 + (y+2)^2$, and we obtain

$$x = \frac{4000}{(y+2)^2} + 1\,. \tag{3}$$

By similar manipulations of the second equation we obtain

$$x = \frac{1000}{(z+3)^2} + 1\,. \tag{4}$$

Note that both (3) and (4) are consistent with the hypothesis that $x > 1$, $y > 1$, and $z > 1$.

By (3) and (4) we have

$$\frac{4000}{(y+2)^2} = \frac{1000}{(z+3)^2} \iff \left(\frac{y+2}{z+3}\right)^2 = 4\,,$$

and since $\frac{y+2}{z+3} > 0$ we have $\frac{y+2}{z+3} = 2$ and $y = 2z + 4$.

Next, we write

$$\begin{aligned} Q(x,y,z) &= xyz + 3xy + 2zx - yz + 6x - 3y - 2z \\ &= (xyz + 3xy + 2xz + 6x) + (-yz - 3y - 2z) \\ &= Q_1(x,y,z) + Q_2(x,y,z)\,. \end{aligned} \tag{5}$$

We have $Q_1(x,y,z) = x(yz + 3y + 2z + 6)$. Substituting $y = 2z + 4$ yields $Q_1(x,y,z) = 2x(z+3)^2$, and then by (4) we obtain

$$Q_1(x,y,z) = 2000 + 2(z+3)^2\,. \tag{6}$$

Next we substitute $y = 2z + 4$ into $Q_2(x,y,z) = -yz - 3y - 2z$ to obtain

$$Q_2(x,y,z) = 6 - 2(z+3)^2\,. \tag{7}$$

By virtue of (5), (6), and (7) we have $Q(x,y,z) = 2006$.

Thus, the expression $Q(x,y,z)$ has a fixed value, namely 2006, so the set of all possible values of $Q(x,y,z)$ is the singleton set $\{2006\}$.